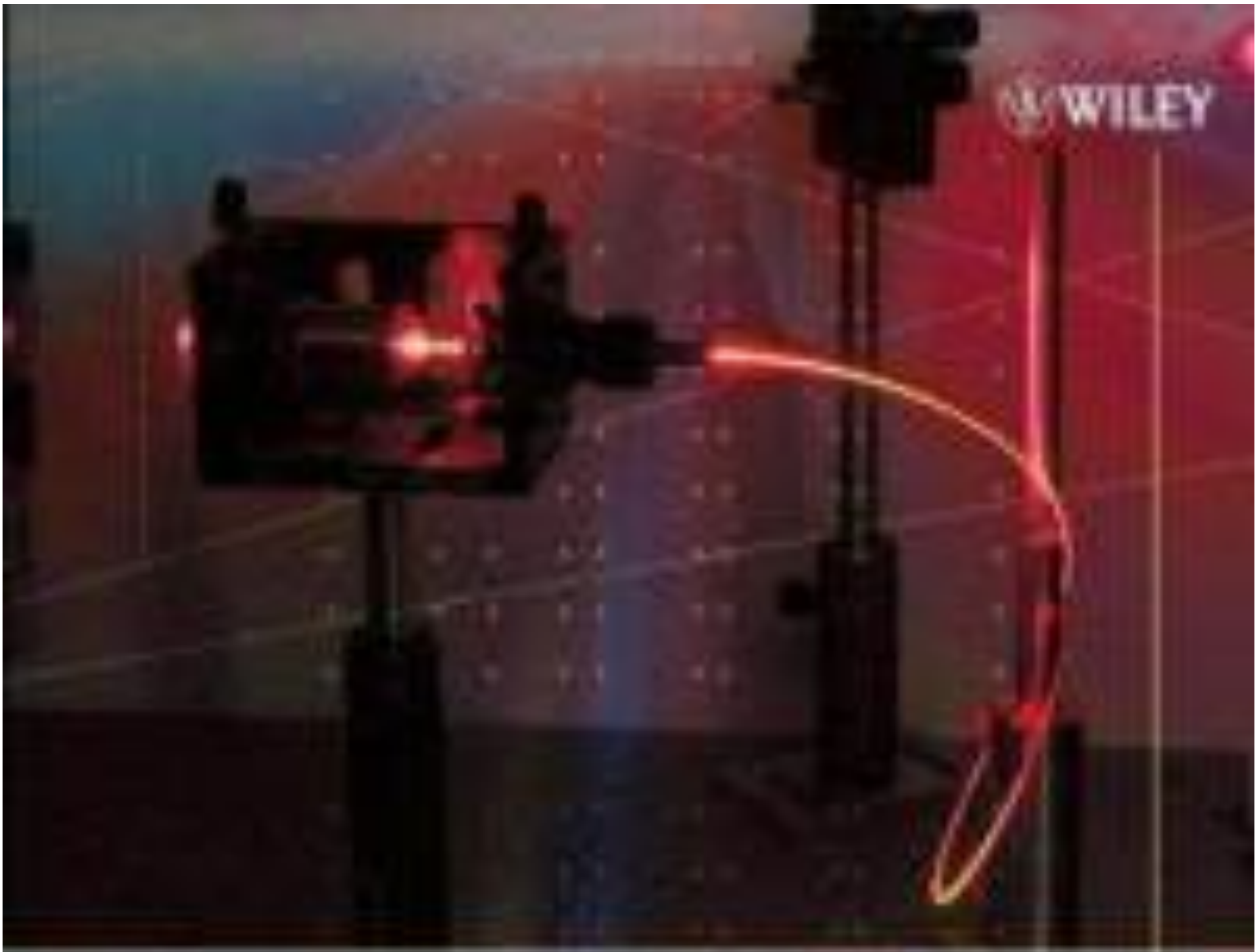

PHOTONICS AND LASERS

AN INTRODUCTION

RICHARD S. QUIMBY

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An Introduction

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Chapter 16

Optical Resonators

One of the three major components of a laser is the optical feedback mechanism, consisting of mirrors or other reflective elements. It is this optical feedback, in combination with optical amplification from stimulated emission, that gives rise to coherent laser oscillations. The simplest arrangement for optical feedback is a pair of mirrors on either side of the gain medium, forming an optical cavity, or optical resonator. The importance of the optical resonator goes beyond simply providing feedback, however. In this chapter, we explore in some detail the effect that the optical resonator has in shaping the frequency spectrum of the emitted laser light.

16-1. MODE FREQUENCIES

Consider the simplified view of an optical resonator shown in Fig. 16-1, with two mirrors separated by a distance L. In general, light can propagate in any direction in between the mirrors, but light that does not propagate close to the resonator axis (i.e., perpendicular to the mirror surfaces) is soon lost from the resonator and is not effective in providing optical feedback. To a first approximation, then, the optical resonator can be analyzed by considering waves only in one dimension.

1-D Treatment

Taking the resonator axis to be in the x direction, we consider electromagnetic plane waves that propagate between the mirrors in the form of Eq. (2-3), $E(x, t) = E_0 \cos(kx - \omega t)$. If the mirrors are highly reflecting, a wave starting at position A will be reflected back and forth between the mirrors many times, and the total E field at the point A will be determined by the superposition, or interference, of the E fields from the many different reflected waves. In general, the phase of the various reflected waves will be different when they reach point A, and the superposition gives rise to *destructive interference.* This levels off the peaks and valleys of the E field distribution, leading to a uniform light intensity within the cavity. However, if the phase of the E field is the same after propagating the round-trip distance $2L$, that is, if

$$E(x + 2L, t) = E(x, t) \tag{16-1}$$

then the reflected waves will reinforce one another, resulting in *constructive interference.* Since the cos function has a periodicity of 2π, this condition is equivalent to

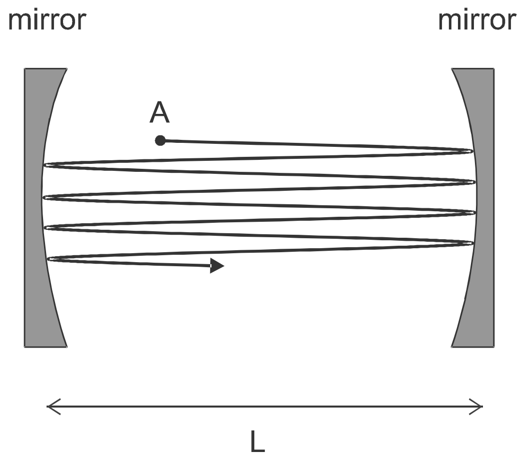

Figure 16-1 Optical cavity of length L with nearly flat mirrors.

$$k\,2L = m2\pi$$
$$\frac{2\pi}{\lambda}\,2L = m2\pi \tag{16-2}$$
$$2L = m\lambda$$

where m is an integer and λ is the wavelength of light in the medium. This equation says that for the waves to add constructively, an integer number of wavelengths must fit into the round-trip distance $2L$. This makes sense physically, since the wavelength is the repeat distance for the traveling wave. The optical frequencies that give constructive interference are then

$$\nu_m = \frac{c/n}{\lambda} = m\frac{c}{2nL} \qquad \text{(mode frequencies)} \tag{16-3}$$

where n is the refractive index of the medium inside the cavity.

At the frequencies given by Eq. (16-3), the reinforcement of the many reflected waves gives rise to a large E field amplitude inside the cavity. This increased amplitude due to multiple reflections is termed resonant enhancement, and the frequencies at which it occurs are called the *resonant frequencies* or *mode frequencies* of the cavity. The physical significance of the mode frequencies is that optical power can be stored in the laser cavity only at these particular frequencies. According to Eq. (16-3), the mode frequencies are all multiples, or harmonics, of a base frequency $c/(2nL)$. The frequency distribution of optical power stored in the resonator cavity is then a "comb spectrum," as illustrated in Fig. 16-2, with the mode frequencies evenly spaced by $c/(2nL)$.

At the resonant frequencies, there are traveling waves moving both left and right in the cavity, which combine to give the total E field at each point. The waves moving in the $+x$ and $-x$ directions can be written as

$$\begin{aligned} E_+(x, t) &= E_0 \cos(kx - \omega t + \phi) \\ E_-(x, t) &= E_0 \cos(kx + \omega t + \phi) \end{aligned} \tag{16-4}$$

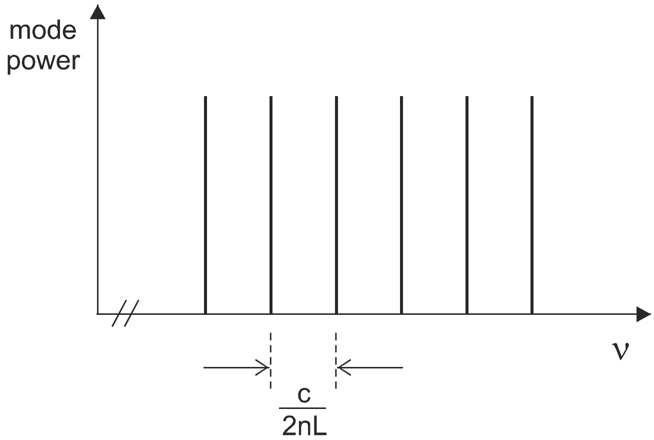

Figure 16-2 Power spectrum for light in a resonant cavity of length L.

where ϕ is a phase constant chosen to match the boundary conditions at the mirrors (for example, $E = 0$ at a metallic mirror). Using the trigonometric identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$ it can be shown that (see Problem 16.1)

$$\begin{aligned} E(x, t) &= E_+(x, t) + E_-(x, t) \\ &= 2\, E_0 \cos(kx + \phi)\cos(\omega t) \end{aligned} \tag{16-5}$$

The spatial and temporal dependence of E given in Eq. (16-5) is that of a *standing wave,* as illustrated in Fig. 16-3.

At a particular value of x, the motion varies in time as $\cos \omega t$, with an amplitude given by $2\, E_0 \cos(kx + \phi)$. The amplitude becomes zero at certain locations, known as *nodes.* At the nodes, the E field and associated electromagnetic energy density ρ (Eq. 2-9) are both zero at all times. For a mode number m, there are $m - 1$ nodes between the cavity mirrors.

EXAMPLE 16-1

Estimate the mode number and mode spacing for an Ar ion laser oscillating at 514 nm in a cavity of length 1 m. Assume $n = 1$.

Solution: The frequency of the laser light is

$$\nu = \frac{3 \times 10^8}{514 \times 10^{-9}} = 5.84 \times 10^{14}\ \text{Hz}$$

and the mode spacing is

$$\frac{c}{2L} = \frac{3 \times 10^8}{(2)(1)} = 1.5 \times 10^8\ \text{Hz}$$

The mode number is then

$$m = \frac{5.84 \times 10^{14}}{1.5 \times 10^8} \simeq 3.89 \times 10^6$$

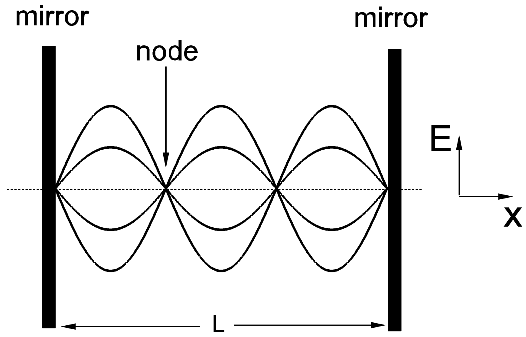

Figure 16-3 Standing wave pattern in laser cavity for $m = 3$, showing $E(x)$ at four values of t.

3-D Treatment

The mode spacing for a three-dimensional cavity can be obtained by extending the 1-D analysis of the previous section. We save for the next chapter a detailed discussion of the stable resonator modes in a laser cavity, and focus here on the general problem of finding the frequencies of modes in an enclosed cavity. Taking the cavity to be a cube of side L, the plane waves that can propagate in the cavity have a wave vector **k** with three components k_x, k_y, and k_z. The condition of Eq. (16-2) now applies to each of these components separately:

$$\begin{aligned} k_x &= m_x \frac{\pi}{L} \\ k_y &= m_y \frac{\pi}{L} \\ k_z &= m_z \frac{\pi}{L} \end{aligned} \tag{16-6}$$

where m_x, m_y, and m_z are positive integers. Negative integers represent the same mode as the corresponding positive integer, because a given mode consists of the combination of traveling waves moving in opposite directions. The different modes can be represented as points in a three-dimensional "k space," as shown in Fig. 16-4, with a spacing between points of π/L. The density of modes is then $(L/\pi)^3$ modes per unit volume of k space. It will prove useful to obtain an expression for the mode density in frequency space for three diminsions. To do this, we note that the frequency ν is related to the magnitude of the wave vector k by $k = 2\pi\nu/c$. For notational convenience, we will let $n = 1$ in the following discussion. In a medium with index of refraction n, the formulae can be generalized by making the substitution $c \to c/n$. The procedure will be to count the number of modes having frequency less than some value ν, and from this to determine the number of modes in a small range of frequencies $d\nu$ around ν.

The number of modes with frequencies up to some value ν is the same as the number of modes with wave vector magnitudes up to a value $k = 2\pi\nu/c$. The surface in k space

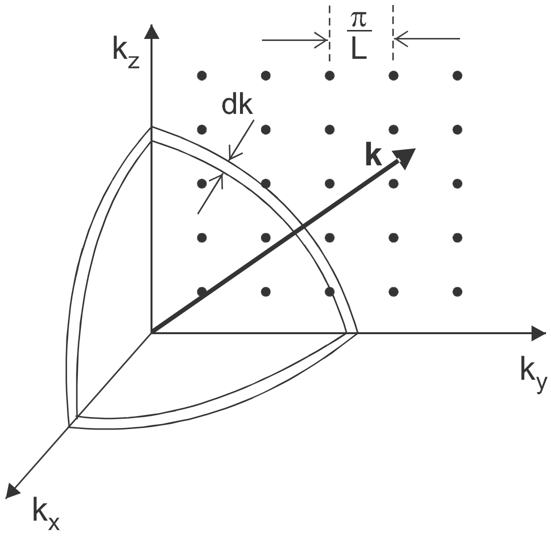

Figure 16-4 Modes in k space.

corresponding to the maximum k value is a sphere of radius k, centered at the origin. The number of distinct modes inside this sphere is then

$$N = \left(\frac{L}{\pi}\right)^3 \times \frac{4}{3}\pi k^3 \times \frac{1}{8} \times 2 \tag{16-7}$$

where the first factor is the number of modes per volume of k space, the second factor is the volume of a sphere of radius k in k space, the factor of 1/8 comes from only considering points with positive k_x, k_y, and k_z, and the factor of 2 comes from the two possible polarizations for each spatial mode. The number of modes having frequency between 0 and ν is then

$$N = \frac{8\pi\nu^3}{3c^3} L^3 \tag{16-8}$$

where $L^3 = V$ is the physical volume of the cavity.

The many modes counted in Eq. (16-8) span a very large frequency range, and most do not interact with the atoms in a laser cavity. The most relevant quantity is the number of modes contained within a small frequency interval $d\nu$ about the center frequency ν. The *spectral mode density* $\beta_\nu(\nu)$ is defined as the number of modes per unit frequency interval, per unit volume V, which from Eq. (16-8) is

$$\begin{aligned} \beta_\nu(\nu) &\equiv \frac{1}{V}\frac{dN}{d\nu} \\ &= \frac{8\pi\nu^2}{c^3} \end{aligned} \tag{16-9}$$

In a small frequency interval $\Delta\nu$, the number of cavity modes is then $\Delta N \approx \beta_\nu V \Delta\nu$. This result will prove to be useful in Chapter 18 when we consider the interaction of atoms with the modes in a laser cavity.

16-2. MODE WIDTH

In the preceding analysis of 1-D resonator modes, we assumed that the modes were perfectly sharp, with well-defined frequencies given by Eq. (16-3). In practice, there is always some spectral broadening of the modes, due to the finite reflectivity of the mirrors. In optics textbooks, the spectral shape of the modes between two parallel mirrors is usually derived by considering the interference of the many reflected beams. For understanding the properties of laser cavities, however, more physical insight can be obtained by considering the time dependence of light intensity in the cavity, and then relating this to the frequency spectrum.

Photon Lifetime

We consider here a laser cavity with no optical gain, which is termed a *passive optical resonator.* Light that happens to be inside the resonator will bounce back and forth between the mirrors, losing energy at each bounce. The rate at which light intensity decays can be determined by considering the loss of intensity in one round-trip through the resonator. Assume that the light has initial intensity I at point A in the cavity, as shown in Fig. 16-5. After reflecting from the right mirror with reflection coefficient R_2, the intensity is $R_2 I$, and after a further reflection from the left mirror the intensity is $R_1 R_2 I$. The change in intensity in one round-trip distance $2L$ is then

$$\begin{aligned} \Delta I &= I(t + \Delta t) - I(t) \\ &= I(t)[R_1 R_2 - 1] \end{aligned} \tag{16-10}$$

where $\Delta t = 2L/c$ is the round-trip time. In this section we will take $n = 1$ for simplicity, but the results can be generalized by replacing $c \to c/n$ in each formula. The time rate of change in intensity is then

$$\frac{\Delta I(t)}{\Delta t} = -\frac{1 - R_1 R_2}{2L/c} I(t) \tag{16-11}$$

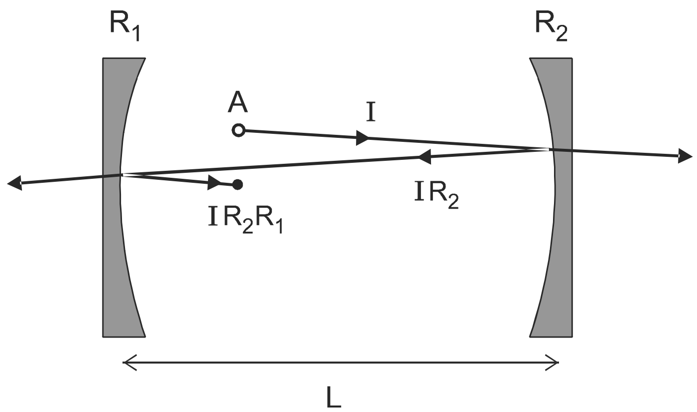

Figure 16-5 Light decreases in intensity during one round-trip through resonator due to mirror reflectivities R_1 and R_2 less than unity.

In laser cavities, the mirror reflectivities are usually high, so the fractional loss per round-trip is $\ll 1$. In this case, $I(t)$ can be approximated as a continuous function, and Eq. (16-11) becomes

$$\frac{dI}{dt} = -\frac{1}{\tau_c} I(t) \tag{16-12}$$

where the *photon lifetime* or *cavity lifetime* τ_c is defined as the time for the light intensity to decay to $1/e$ of its initial value. For small loss per round-trip, we have

$$\tau_c \simeq \frac{2L}{c(1 - R_1 R_2)} \qquad \text{(photon lifetime)} \tag{16-13}$$

The solution of Eq. (16-12) is

$$I(t) = I_0 e^{-t/\tau_c} \tag{16-14}$$

which can be easily verified by substitution. The light intensity in the cavity decays exponentially in time, with a decay time equal to the photon lifetime τ_c. The measurement of this decay time is one method of making accurate determinations of mirror reflectivities close to 1. In the *ring-down technique,* a short pulse is sent into the cavity, and the light exiting the cavity is monitored versus time. Mirror reflectivities are determined from the measured cavity lifetime using Eq. (16-13).

The frequency spectrum of the modes is determined from the time decay using the time–frequency uncertainty relation. The time dependence of E is that of a damped sinusoid,

$$E(t) = E_0 e^{-(t/2\tau_c)} \cos \omega t \tag{16-15}$$

as illustrated in Fig. 16-6. The time constant for the decay of $E(t)$ is $2\tau_c$ because $E \propto \sqrt{I(t)}$ (Eq. 2-9). In Appendix B it is shown that this type of time decay is characterized by the uncertainty relation

$$\Delta\omega_{1/2}\, \tau_c \simeq 1 \qquad \text{(uncertainty relation)} \tag{16-16}$$

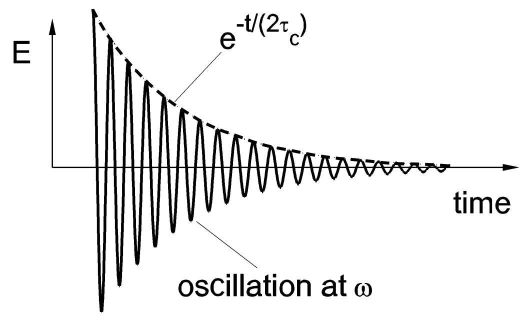

Figure 16-6 The E field oscillations in the cavity at angular frequency ω decay exponentially with time constant $2\tau_c$. In this plot, $\omega\tau_c = 5\pi$.

where $\Delta\omega_{1/2}$ is the angular frequency *full width at half maximum* (FWHM). Using $\Delta\nu_{1/2} = \Delta\omega_{1/2}/(2\pi)$, the frequency width of the modes can be written as

$$\Delta\nu_{1/2} \simeq \frac{1}{2\pi}(1 - R_1R_2)\frac{c}{2L} \qquad \text{(frequency width of mode)} \qquad (16\text{-}17)$$

This expression assumes high mirror reflectivities, and is accurate within ~ 10% for $R_1R_2 \geq 0.80$. An expression valid for lower R is derived in Problem 16.6.

The frequency distribution of light intensity in the laser cavity is illustrated in Fig. 16-7, with the modes of width $\Delta\nu_{1/2}$ separated by $c/(2L)$. Although we have introduced them from the classical physics point of view, these cavity modes can be thought of as quantum states of the electromagnetic field. The photon, which is the quantum of the electromagnetic field, can be though of as "occupying" these cavity mode states, just as an electron occupies various quantum states in an atom or solid. From this viewpoint, the uncertainty relation in Eq. (16-16) becomes the Heisenberg uncertainty principle relating energy and time, $\Delta(\hbar\omega)\,\Delta t \simeq \hbar$. The energy of the photon $\hbar\omega$ is uncertain because it is uncertain when during the time τ_c the photon leaves the cavity.

Quality Factor *Q*

The time decay of the E field in an optical resonator is similar to that of a damped harmonic oscillator, and the terminology that is used to describe the sharpness of a resonance in the damped harmonic oscillator can also be applied to the optical resonator. The *quality factor Q* of a resonance is defined as the center frequency divided by the width, or

$$Q \equiv \frac{\nu}{\Delta\nu_{1/2}} \qquad \text{(quality factor of resonance)} \qquad (16\text{-}18)$$

which can be written here as

$$Q \simeq \frac{\nu(2L)(2\pi)}{(1 - R_1R_2)c} = \frac{4\pi L}{\lambda(1 - R_1R_2)} \qquad (16\text{-}19)$$

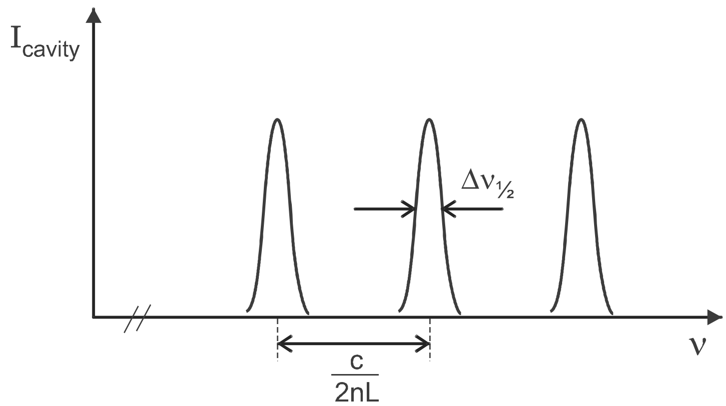

Figure 16-7 Cavity modes have width $\Delta\nu_{1/2}$ and spacing $c/(2L)$.

It is seen from Eq. (16-19) that the relative sharpness of the modes is greatest for very high reflectivity mirrors and long cavity lengths. To achieve very narrow laser linewidths, with correspondingly long coherence times T_c (see Chapter 15), the mirror reflectivities should be high. Most lasers in the visible and near IR regions use special mirrors made with multilayer dielectric thin films, rather than metallic mirrors, because they can be made to have a very high reflectivity over some range of wavelengths. Ordinary aluminum mirrors typically have $R \leq 0.90$, and are not often used in laser resonators.

Cavity Finesse

As the laser cavity length L increases, the modes become narrower, but the spacing between modes also decreases. A useful parameter that gives the mode width compared with the mode spacing is the *finesse,* defined by

$$\mathcal{F} \equiv \frac{\text{mode spacing}}{\text{mode width}} = \frac{c/(2L)}{\Delta\nu_{1/2}} \qquad \text{(finesse of cavity)} \tag{16-20}$$

Using Eq. (16-17), the finesse can be written as

$$\mathcal{F} \simeq \frac{2\pi}{1 - R_1 R_2} \tag{16-21}$$

which is valid for high-reflectivity mirrors. Note that the finesse is independent of the cavity length, depending only on the mirror reflectivities. The finesse and cavity Q can be related using Eqs. (16-3), (16-19), and (16-21), giving

$$Q = \mathcal{F}\frac{\nu}{c/(2L)} = m\mathcal{F} \tag{16-22}$$

where m is the mode number. The three quantities Q, $\mathcal{F}$, and $\Delta\nu_{1/2}$ are thus equivalent ways of describing the spectral width of the cavity modes.

EXAMPLE 16-2

A He–Ne laser cavity is 1 m long with mirror reflectivities of 0.99, and operates at 632.8 nm. Determine the cavity Q, the finesse, the mode number, and the frequency width of a cavity mode in this laser. Assume the index of refraction is $n = 1$. Also, if the laser light were confined to a single cavity mode, what would be the coherence length of the light?

Solution: The frequency of the light is

$$\nu = \frac{3 \times 10^8}{632.8 \times 10^{-9}} = 4.74 \times 10^{14}\ \text{Hz}$$

and the mode number is

$$m = \frac{\nu}{c/(2L)} = \frac{2L}{\lambda} = \frac{2(1)}{632.8 \times 10^{-9}} = 3.16 \times 10^6$$

The finesse is

$$\mathcal{F} = \frac{2\pi}{1-(0.99)^2} \simeq 316$$

and the quality factor is

$$Q = m\mathcal{F} \simeq (3.16 \times 10^6)(316) = 9.98 \times 10^8$$

The mode width can be found by either

$$\Delta\nu_{1/2} = \frac{\nu}{Q} = \frac{4.74 \times 10^{14}}{9.98 \times 10^8} = 4.75 \times 10^5 \text{ Hz}$$

or

$$\Delta\nu_{1/2} \simeq \frac{1}{2\pi}(1 - [0.99]^2)\,\frac{3 \times 10^8}{2(1)} = 4.75 \times 10^5 \text{ Hz}$$

The coherence length is

$$L_c = cT_c = \frac{c}{\Delta\nu_{1/2}} = \frac{3 \times 10^8}{4.75 \times 10^5} \simeq 630 \text{ m}$$

Ordinary He–Ne lasers oscillate on more than one mode, and the coherence length is much less than this.

16-3. FABRY–PEROT INTERFEROMETER

In the previous sections, we considered a pair of mirrors as a way of providing feedback for a laser, confining light inside the optical cavity. Another application for such a resonator is to act as an optical frequency filter, in which case it is called a *Fabry–Perot interferometer.* In this application, light of intensity I_{in} is incident externally on one side of the resonator, as in Fig. 16-8, and after multiple reflections within the cavity, light of intensity I_{out} exits through the other side. The transmission efficiency is defined as $T = I_{out}/I_{in}$, and varies with frequency as shown in Fig. 16-9. At the mode frequencies for the cavity, nearly all of the incident light is transmitted ($T \simeq 1$), whereas for frequencies off resonance very little light is transmitted. The full width at half maximum (FWHM) of each transmission peak is $\Delta\nu_{1/2}$ [Eq. (16-17)], which becomes very small for high-reflectivity mirrors. In effect, the Fabry–Perot is a narrow band pass optical filter, with a regular array of transmission peaks spaced by $c/(2L)$, giving a comb-shaped frequency spectrum.

It may seem puzzling at first that the transmission of the Fabry–Perot interfermometer can be 100% when the mirror reflectivites are very high, since these high mirror reflectivites should prevent most light from passing through. The resolution to this apparent paradox is to realize that at resonance, the light intensity inside the cavity builds up to a value much higher than that of the incident light. For example, if $R_1 = R_2 = 0.99$, then only 1% of the light I_{cav} circulating inside the cavity is transmitted through the output mirror R_2

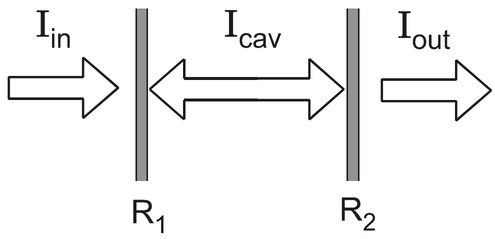

Figure 16-8 Transmission I_{out}/I_{in} through a Fabry–Perot interferometer is high at resonance, where the optical intensity I_{cav} inside the cavity builds up due to constructive interference of the multiple reflections.

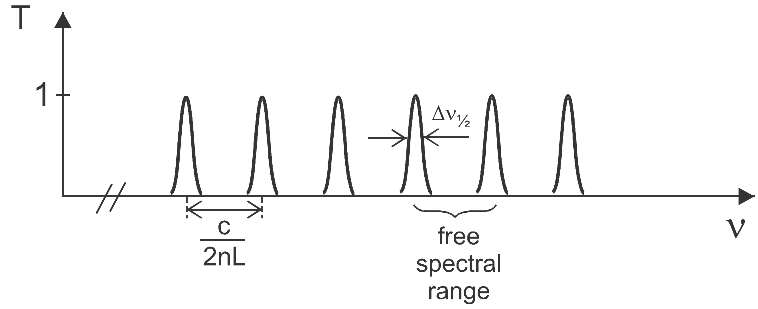

Figure 16-9 Transmission spectrum of the Fabry–Perot interferometer.

in each bounce. But if the light intensity inside the cavity at resonance builds up to a value $I_{cav} \simeq 100\, I_{in}$, then $I_{out} = (0.01)\, I_{cav} \simeq I_{in}$. Off resonance, the intensity inside the cavity remains low, and $T \ll 1$.

One application of the Fabry–Perot interferometer is in high-resolution optical spectroscopy. Fig. 16-10 shows a representative *fluorescence spectrum* (frequency distribution of emitted light) for an atomic transition, along with the Fabry–Perot transmission peaks in the vicinity of the fluorescence. If the emitted light is made to pass through the Fabry–Perot interferometer before detection, the detected signal will be the product of the fluorescence intensity I_f and the Fabry–Perot transmission T. The detected signal then corresponds to the part of the fluorescence spectrum that lines up with one or more of the cavity modes of the Fabry–Perot interferometer. The mode frequencies can be tuned continuously by varying L with a piezoelectric transducer, and the fluorescence spectrum can then be mapped out by scanning a single mode across the spectrum.

Light from different parts of the spectrum may be detected simultaneously in different *orders,* or mode numbers m, depending on the width of the fluorescence spectrum compared with the mode spacing $c/(2L)$. Since the mode spacing is the frequency range over which there are no interfering orders in the measured spectrum, it is also referred to as the *free spectral range.* If the medium between the mirrors has a refractive index n, the free spectral range is $c/(2nL)$. The mode frequencies can be swept by changing n as well as L.

EXAMPLE 16-3

A Fabry–Perot interferometer uses mirrors with reflectivity 0.99 spaced by 1 mm, with an air gap between them. (a) Determine the frequency resolution when measuring the sodium "D" spectral line at 589 nm. (b) Over what wavelength range is the measured spectrum free from overlapping orders?

Solution:

(a) From Eq. (16-17) the mode width is

$$\Delta\nu_{1/2} \simeq \frac{1}{2\pi}(1-[0.99]^2)\frac{3\times 10^8}{2(1\times 10^{-3})} = 475 \text{ MHz}$$

(b) The free spectral range is

$$\Delta\nu_{\text{FSR}} = \frac{3\times 10^8}{2(1\times 10^{-3})} = 1.5\times 10^{11}\text{ Hz} = 150\text{ GHz}$$

In terms of wavelength this is

$$\Delta\lambda_{\text{FSR}} = \frac{\lambda^2}{c}\,\Delta\nu_{\text{FSR}} = \frac{(589\times 10^{-9})^2}{3\times 10^8}(1.50\times 10^{11}) = 1.73\times 10^{-10}\text{ m}$$

This Fabry–Perot interferometer can, therefore, only be scanned over 0.173 nm before overlapping orders appear in the spectrum. This example illustrates both the advantages and disadvantages of the Fabry–Perot interferometer. Very high resolution can be obtained, but at the expense of a limited scanning range.

PROBLEMS

16.1 Show that Eq. (16-5) follows from Eq. (16-4) using the trigonometric identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

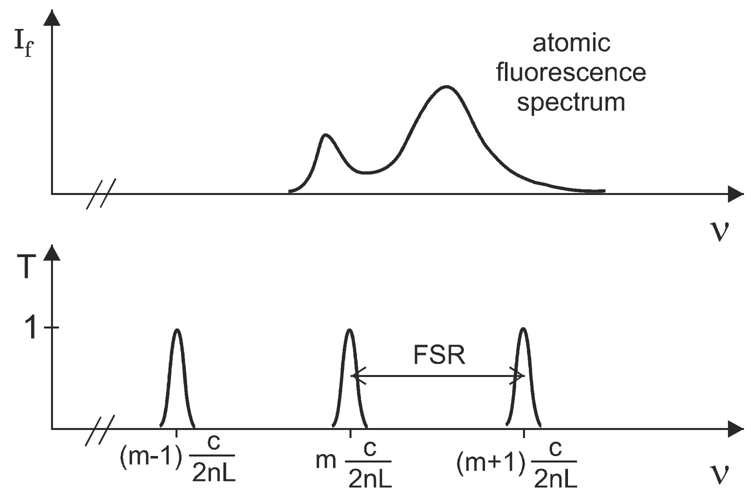

Figure 16-10 The Fabry–Perot interferometer can be used to analyze an optical frequency spectrum when the mode frequencies are continuously varied.

16.2 A GaAs diode laser has a cavity formed by the Fresnel reflections from the end facets of the GaAs chip, which is 0.8 mm long. If the laser wavelength (in air) is $\approx$ 850 nm, determine the approximate mode number and the mode spacing (in nm) for this laser.

16.3 A He–Ne laser cavity has a spacing of 15 cm between the mirrors, and the optical mode in the cavity has a diameter of $\approx$ 3 mm. (a) Determine the frequency difference between adjacent laser modes. (b) Determine the frequency difference between all possible cavity modes contained within the laser cavity volume. (c) The He–Ne gas mixture provides optical gain over a frequency width of $\approx$ 1.5 GHz. Compare the number of laser modes that are within this width to the total number of cavity modes within this width.

16.4 A ring-down measurement is made on an optical cavity with two identical high-reflectivity mirrors spaced by 45 cm in air. When a short pulse is sent into the cavity, the pulse intensity is observed to decay to 20% of its initial value in a time of 806 ns. Determine the mirror reflectivity to three significant figures.

16.5 In deriving Eq. (16-13) for the photon lifetime, it was assumed that the fractional loss per round trip is small, that is, $1 - R_1R_2 \ll 1$. If this condition does not hold, an alternative expression can be obtained that is valid for smaller R. (a) Show that $(R_1R_2)^p$ is the fraction of light remaining in the cavity after p complete round-trips, and set this equal to e^{-1} to show that the cavity lifetime (time required for the light to decay to e^{-1} of its initial value) is

$$\tau_c = \frac{2nL/c}{\ln(1/R_1R_2)}$$

(b) Show that this reduces to Eq. (16-13) for $R_1R_2 \approx 1$. (c) What is the percentage difference between the two expressions for τ_c when $R_1R_2 = 0.8$?

16.6 Using the expression for cavity lifetime from Problem 16.5, derive expressions for mode width, cavity Q, and cavity finesse that are valid for small R.

16.7 A semiconductor cavity is formed by cleaving the ends of a semiconductor chip so they are nearly parallel. Instead of external mirrors, the cavity relies on Fresnel reflection from the semiconductor–air interface. Assume an index of refraction 3.5, cavity length 0.8 mm, and laser wavelength 830 nm. (a) Use the results of Problem 16.5 to calculate the spacing and width of the longitudinal cavity modes (both in frequency and in wavelength). (b) Use the results of Problem 16.6 to calculate the Q and finesse of the cavity.

16.8 The semiconductor laser of Problem 16.7 is now modified to use external mirrors for the optical cavity. One mirror has $R = 0.98$, the other mirror has $R = 0.95$, and they are deposited directly on the ends of the semiconductor chip. Determine (a) the spacing between cavity modes, (b) the spectral width of the cavity modes, and (c) the cavity finesse.

16.9 Consider a variation of the laser of Problem 16.8, in which the two mirrors are freestanding and separated by 5 cm in air. The semiconductor (still of length 0.8 mm) between the mirrors is slightly tilted so that any Fresnel reflection from the semiconductor–air interface is lost from the cavity. Determine (a) the spacing between cavity modes, (b) the spectral width of the cavity modes, and (c) the cavity finesse.

16.10 An air-spaced Fabry–Perot interferometer has mirror spacing 0.15 mm and mirror reflectivities $R = 0.99$. It is used to measure the spectrum of the sodium doublet, which consists of two closely spaced emission lines at 588.995 and 589.592 nm. (a) Determine the mode number of the FP resonance. (b) By how much must the plate spacing be changed in order to scan a single mode from one of the lines to the other? (c) By how much must the plate spacing be changed so that the same emission line is seen again in a different order? If the plate spacing was increased, is the new mode number higher or lower? (d) What is the wavelength resolution of the resulting spectrum?